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STRATEGIC BIDDING WITH REGARD TO DEMAND ELASTICITY*

S. SOLEYMANI[1**], A. M. RANJBAR[1,2] AND A. R. SHIRANI[2]
[1]Dept. of Electrical Engineering, Sharif University of Technology, Tehran, I. R. of Iran
Email: ssoleymani@mehr.sharif.edu
[2]Niroo Research Institute, Tehran, I. R. of Iran

Abstract– This paper presents a new method to analyze the bidding strategies of Generating Companies (GENCOs) with regard to demand elasticity. It is assumed that the available information of each GENCO about its opponents is incomplete and only the minimum and maximum generation levels of their opponents, as well as their fuel type, are known. In the proposed methodology, GENCOs prepare their strategic bids according to a Supply Function Equilibrium (SFE) model. GENCOs will change their bidding strategies until Nash equilibrium points are obtained. The general Algebraic Modeling System (GAMS) has been used to solve the maximization modules using the MINOS optimization software with non Linear Programming (NLP).

Keywords– Bidding strategy, Demand elasticity, GENCOs, Nash equilibrium point

1. INTRODUCTION

Recent changes in the electricity industry in several countries have led to a less regulated and more competitive energy market. In this condition, cost is replaced by price and each GENCO will try to maximize its own profit. For a GENCO, it is critical to devise a good bidding strategy according to its opponents' bidding behavior, the model of demand and power system operating conditions.

Generally there are different methods for developing bidding strategies in electricity markets. A non-cooperative incomplete game was employed in [1] and [2] to choose a GENCO's optimal bidding strategy in deregulated power pools. Each pool participant knew its own operation costs, but didn't know his or her opponent's costs. The game with incomplete information was transformed into a game with complete, but imperfect information and was solved using the Nash equilibrium idea. In [3], competitors' bids were known and the authors performed an optimization procedure to find the Nash equilibrium based on the bid sensitivity of these competitors. In [4], the genetic algorithm was used to develop a bidding strategy for the generator and distributor during the trading process. In [5], the bidding problem was modeled as a bi-level problem by assuming complete information on GENCO's opponents. The papers [6-8] have studied the equilibrium production of the generation companies from different aspects using the game model. Zu *et al*. [7] studied the impact of price caps on the electricity market. A method to predict the optimal energy production of a power producer in an oligopoly electricity market is presented in [6]. However this model doesn't consider the technical constraints of the generation companies. In [9], a modified cournot, non-cooperative game model is used to determine the expected equilibrium state of the oligopoly electricity market. Each generation company knows the market inverse demand function and has several estimated cost functions for each generation unit of the other companies.

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**Corresponding author

A stochastic optimization method is proposed in [10] based on the Monte Carlo method to find generators' optimal bidding strategies. A detailed literature review of bidding strategies in electricity markets is presented in [11]. Rodriguez and Anders have presented a methodology to design an optimal bidding strategy for a generator according to its degree of risk aversion, the forecasted price and the probability distributions of errors in the forecasted price for each hour in a day [12]. With the forecasted price, the profit maximization is performed to find the optimal production and consequently, the bidding curve is obtained.

Tao li *et al*. [13] have extended the proposed methods in [1] and [2] for developing a more general approach to GENCOs' optimal bidding strategies with incomplete information in the electricity markets. The proposed methodology employs the supply function equilibrium for modeling a GENC'O bidding strategy. The competition is modeled as a bi-level problem with the upper sub-problem representing the individual GENCO, and the lower sub-problem representing the ISO (Independent System Operator).

In this paper, GENCOs prepare their strategic bids according to the SFE model and with regard to price-dependent demand.

When demand depends on price, if GENCOs choose a high price, it causes an increment in the Market Clearing Price (MCP), and as a consequence, the decrement of demand. In this condition, GENCOs should try to find the Nash equilibrium points for their bidding strategies.

We suppose that all GENCOs are intelligent and have knowledge of demand function. Therefore, finding the Nash equilibrium points as soon as possible is important. In order to reach this target, a good estimate of initial bidding and knowledge about the degree of risk aversion of GENCOs may speed up the convergence of the proposed algorithm.

This paper is organized as follows: Section II is problem formulation. This section includes modeling of demand, estimating opponents unknown information, GENCO's bids, and a market clearing model. The proposed solution method is shown in Section III. Section IV gives an illustrative example with three GENCOs. The discussion on the proposed model is given in Section V, and finally, Section VI provides the conclusion.

2. PROBLEM FORMULATION

a) Modeling of demand

Economists believe that a market will not be real until demand elasticity has been considered. So in order to reach the optimal bidding strategies of GENCOs in the actual market, the demand function is created on the basis of expression (1)

$$\rho = k\, q^{\frac{1}{\beta}} \tag{1}$$

Where ρ is price, q is quantity, k is a calibration constant and β is a negative number that shows the elasticity of demand. Function (1) is a Cobb-Douglas function, which is iso-elastic. This means that elasticity, defined as the relative demand change divided by the relative price change, is constant throughout the curve.

To simplify the market simulation model, we assume that the electricity demand function is a strictly decreasing linear function of the price ρ. This function is expressed as:

$$\rho = k_1 - k_2 q \qquad (k_1 > 0 \;\; \& \; k_2 > 0) \tag{2}$$

In this function, k_2 shows the elasticity of demand. Figure 1 shows this function when the demand elasticity changes.

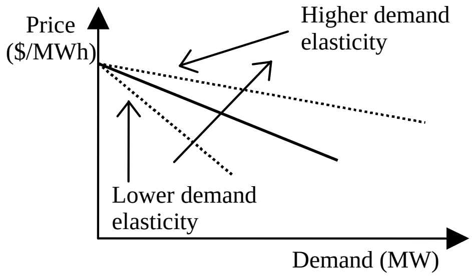

Fig. 1. Demand function model

b) Estimating opponents unknown information

It is necessary for a GENCO to model its opponents' unknown information. If we suppose that all GENCOs have only thermal units, the most important parameters for GENCOs will be a, b, and c coefficients of the generating cost function ($a p^2 + b p + c$). All GENCOs try to hide this information from the others, so opponents should estimate them based on the available information.

The available information of GENCOs about their opponents is incomplete and only the minimum and maximum generation levels of their opponents, as well as their fuel types are known.

Elhawary presented a method to obtain the fuel cost curve as a quadratic function of active power generation [14]. This function is expressed as:

$$F(P) = \alpha P^2 + \beta P + \gamma \tag{3}$$

In this function F(P) is measured in MJ/h or MBtu/h, so by considering the High Heat Value (HHV) of fuels and the fuel price(in $\$/m^3$ or $/lit), F(P) is obtained in $. Therefore the fuel price should be forecasted for future times in order to obtain the fuel cost. We can define several scenarios with definite probability for the fuel price, so a different type for α , β and γ coefficients will result.

The total cost of operation includes the fuel cost, cost of labor, supplies and maintenance. These costs, with the exception of fuel cost, are expressed as a fixed percentage of the fuel costs, so the total generating cost function ($a p^2 + b p + c$) will be a factor of the fuel cost function, and a, b, and c coefficients will be calculated as a factor of α , β , and γ .

c) GENCO's bids

In a power market, GENCOs may prepare their strategic bids according to the four models in imperfect competition. These models are Bertrand, Cournot, Stackelberg and SFE where the Stackelberg model is similar to the Cournot model [15].

Figure 2 illustrates where the intensity of competition predicted by the basic formulation of each of the models places them along the competitive spectrum.

Among these models, only SFE enables a GENCO to link its bidding price with the bidding quantity of its product and only this model is the closest to the actual behavior of players in the actual power market. In this paper, we suppose that GENCOs use an SFE model to prepare their strategic bids. The bid function of a unit is expressed as:

$$\rho_j = \mu_j q_j + MC_j \tag{4}$$

where

ρ_j : The offered price of unit j

q_j : The quantity corresponding to ρ_j

μ_j : Mark-up coefficient of unit j

MC_j : Marginal cost of unit j

The generating cost function is a function of active power generation and is expressed as:

$$C_j = C(q_j) = a_j.q_j^2 + b_j.q_j + c_j \quad (j = 1,2,...,n_i) \tag{5}$$

where a_j , b_j , and c_j are the coefficients of generating cost function.
The marginal cost of each unit is expressed as:

$$MC_j = 2\,a_j q_j + b_j \tag{6}$$

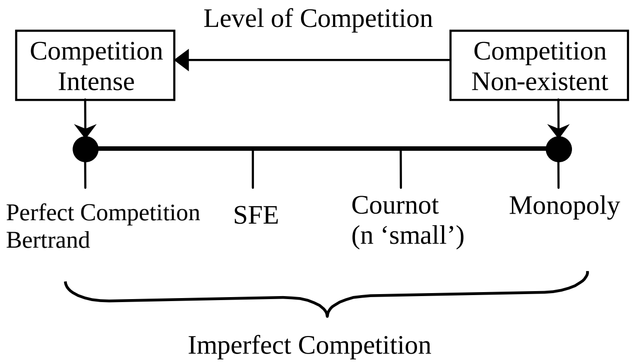

Fig. 2. Equilibrium models and predicted degree of competition [15]

The real power markets in the world are not characterized by perfect competition, but rather an oligopoly market, i.e. a market in which there is not only one player (a monopoly) and not an infinite amount of players (perfect competition). In an oligopoly market, if all GENCOs bid equal to their marginal cost, the market power will not produce. So the exercise of market power by each player is used as an opportunity to add a mark-up to the player's own resultant supply function. A player will behave as a price taker in the market if the mark-up coefficient is 0.

We suppose the marginal cost of generators is a piece-wise supply curve. An example of supply curve with and without a mark-up is shown in Fig. 3.

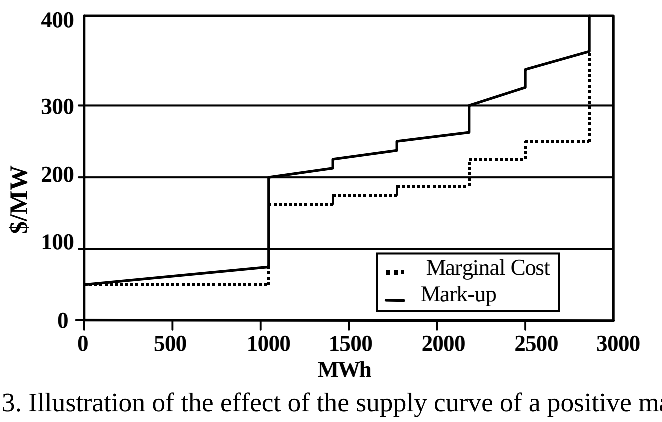

Fig. 3. Illustration of the effect of the supply curve of a positive mark-up

d) Market clearing model

GENCOs will submit bid curves to the ISO, then ISO clears the market after collecting bids. In the ISO's market clearing model, ISO dispatches units in the order of the lowest to the highest bid as needed to meet demand considering network constraints. The bid price of the last unit dispatched sets the market clearing price, then all units dispatched receive the same market clearing price regardless of the unit bid price. In this paper, the maximization of social welfare is considered as the objective function, which ISO is going to solve. The social welfare is the sum of consumer and producer surplus. Consumer surplus is the utility or benefit the consumer gains from a given equilibrium point. Similarly, producer surplus is the benefit or profit the producer gains from a given equilibrium point. An example of supply, demand, equilibrium point and producer and consumer surplus is shown in Fig.4.

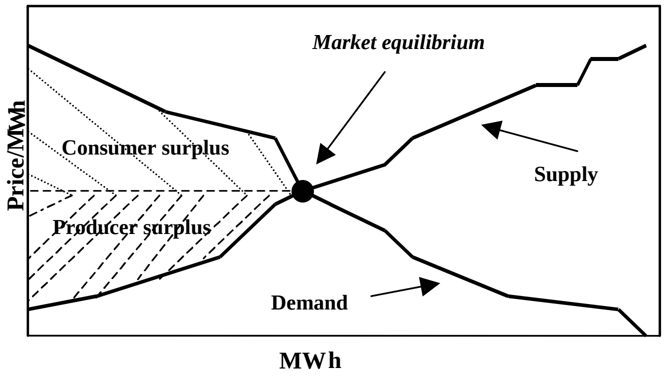

Fig. 4. Supply, demand, price equilibrium and producer and consumer surplus

The optimization model of market clearing can be expressed as follows:

$$\begin{aligned}
&\max \quad q_d * \rho_d - \sum_{j=1}^{N_j}\sum_{b=1}^{N_B} q_{jB}^{G} * \rho_{jB}^{Gbid} \\
&\textit{subject to}: \\
&\quad q_{jB\min} \le q_{jB}^{G} \le q_{jB\max} \\
&\quad \rho_d = k1 - k2 * q_d \\
&\quad \sum_{j=1}^{N_j}\sum_{b=1}^{N_B} q_{jB}^{G} = q_d \\
&\quad q_{d\min} \le q_d \le q_{d\max}
\end{aligned} \tag{7}$$

where

N_B: Number of blocks of the bid function for every unit.

N_j: Number of units in the market.

K1, k2: the coefficients of demand model.

q_d : The system demand

ρ_d : Price corresponding to the demand q_d

q_{jB}^{G} : Power produced with the b-th block of unit j

ρ_{jB}^{Gbid} : Price corresponding to the b-th block of unit j

$q_{jB\min}, q_{jB\max}$: Lower and upper bounds of the b-th block of unit j

$q_{d\min}$, $q_{d\max}$: The lower and upper bounds of demand

3. FINDING NASH EQUILIBRIUM POINTS

As shown in Fig.4, when GENCOs change their bid curves, the aggregated bid supply curve, and then the market equilibrium point, will be changed. If GENCOs increase their mark-up coefficients, the aggregated bid supply curve will be replaced to the upper level and in spite of price increment, the market equilibrium point shows demand decreasing.

In this condition, GENCOs may or may not obtain more profit than their previous offers, therefore each GENCO should choose the best mark-up strategy to obtain the maximum profit based on the other player's behavior, demand model and technical constraints.

The solution to this problem is to obtain Nash equilibrium points. At Nash equilibrium points, each player's strategy is the best response to the other player's strategies that are actually played. Therefore, neither player has an incentive to change its strategy [15].

Mathematically it can be shown as follows:

$$\begin{aligned} &\forall\, i \in I, \mu_i \in S_i \\ &\Pi_i(\mu_1^*, \mu_2^*, \ldots, \mu_i^*, \ldots, \mu_n^*) \geq \Pi_i(\mu_1^*, \mu_2^*, \ldots, \mu_i, \ldots, \mu_n^*) \end{aligned} \tag{8}$$

where

Π_i : The profit of the ith producer that depends on its own strategy and the strategy of other producers.

μ_i : The strategy of the ith producer

μ_i^* : The optimal strategy of the ith producer

S_i : Event space for the ith producer's choice of strategy

I: The set of Producers

Figure 5 shows the proposed algorithm to determine the optimal bidding strategy of GENCOs.

The following iterative procedure is used to determine the Nash equilibrium points:

Initialization $\mu_i = 0, \quad \forall i$

Step 1 μ_1^* is determined so that (8) is satisfied for i=1

Step 2 μ_2^* is determined so that (8) is satisfied for i=2

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Step n μ_n^* is determined so that (8) is satisfied for i=n

Steps 1 to n are repeated until the equilibrium is determined.

This game problem may have only one Nash equilibrium, multiple Nash equilibria, or none at all. The upper limit considered for the mark– up strategy of units may also be a factor in multiple Nash equilibria.

The computational requirement for the proposed algorithm will increase with the number of units. Meanwhile, a GENCO or unit may speed up the convergence of the algorithm by providing a good estimate of the initial mark- up strategy, which could be based on the degree of market power ability.

This algorithm can be used to simulate the market power. In connection with the simulation of market power, it is assumed that all major producers wish to maximize their profit. So the simulation of market power is a simulation of the players' profit maximization. Since every one (who has the possibility of exercising market power) is also trying to maximize its own profit function, this can be described as a multi-criteria problem. The Nash equilibrium point is the best response of each player to the others with regard to profit maximization of players. The obtained Nash mark-up strategy of units in the end of the algorithm shows the ability of the market power of the players.

Whenever the Nash mark-up strategy of units is large, the market power ability of these units is greater than the others.

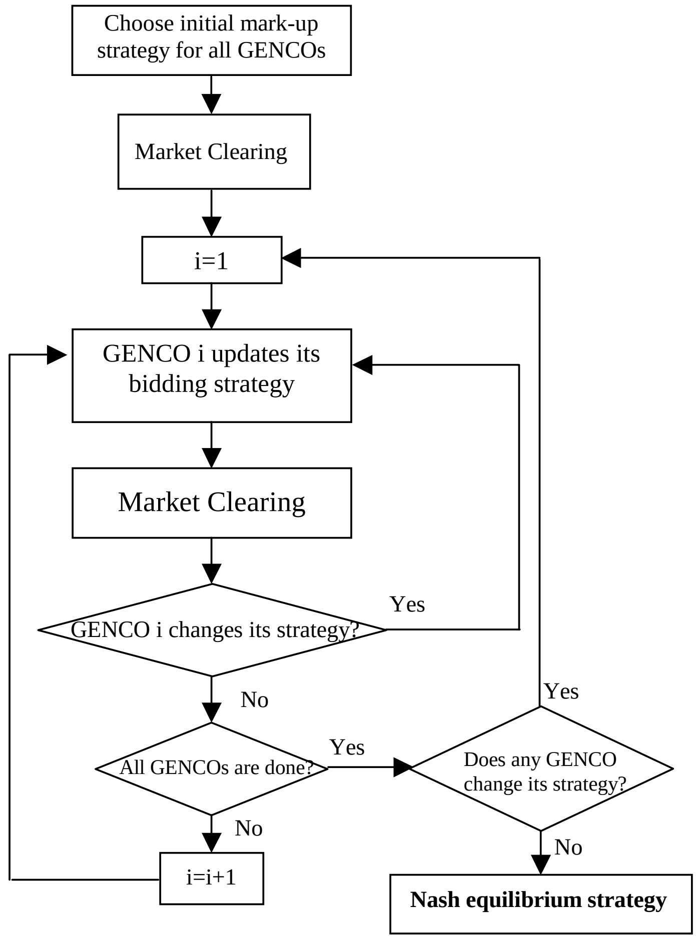

Fig. 5. Procedure of optimal bidding strategy calculation

4. NUMERICAL EXAMPLE

We suppose a market with 3 units and each unit as a GENCO. It is supposed that GENCOs can forecast the exact price of fuel, and we also assume that all of the GENCOs are intelligent and aware of this price. So only one generation cost structure for GENCOs is defined in Table 1.

Figure 6 shows the demand model of this market. As mentioned in Section 2a, demand function is considered a linear function. In this example we assume a minimum and maximum consumption.

Each unit updates its strategy as follows:

$$\mu_i^{new} = \mu_i^{old} + \varepsilon \tag{9}$$

In this equation ε is constant and can be adjusted in each step to speed up the convergence automatically or manually.

The model described in this paper has been written in GAMS (General Algebraic Modeling System) language [16]. GAMS has been used to solve the social welfare maximization module using the MINOS optimization software with non linear programming.

Table 1. Cost coefficients of GENCOs

GENCO	a	b	c	Pmin (MW)	Pmax (MW)
1	0.0025	8.4	225	45	350
2	0.0081	6.3	729	45	350
3	0.0025	7.5	400	47.5	450

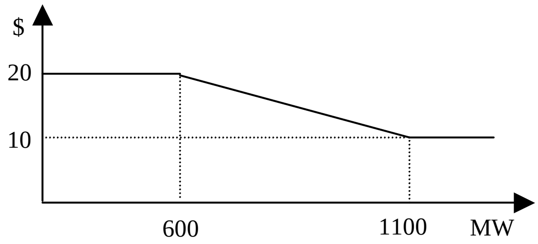

Fig. 6. Demand model

Case 1- In this case, the initial mark up strategy is supposed equal to “0” for all units. The optimal mark-up strategy, the MW dispatched and the expected pay off of units are shown in Table II. In this paper, we suppose that, when the energy market is cleared, each unit will be paid according to the market clearing price, so the pay off for each unit is calculated as follows:

$$R(i) = MCP * q(i) - a(i) * q(i)^2 - b(i) * q(i) - c(i) \quad (10)$$

where
MCP: Market clearing price
$q(i)$: The MW dispatched of unit i
R(i): The obtained pay off for unit i

Table 2. The results of algorithm for each unit in case 1

GENCO	Mark-up strategy	MW dispatched	The expected pay off($)
1	0.005	295	427.7
2	0.0043	240	14.32
3	0.0039	385	711.6

The market clearing price and demand will be equal to (11.35 $/MW) and (920 MW) respectively.

Case 2- In this case, we consider demand is not dependent on the price and is equal to its obtained amount in Case 1, where Table 3 shows its results.

As shown in Tables 2 and 3, the mark- up strategy of units in Case 2 is higher than Case 1. It shows that when consumers have a reaction to the price, it is not profitable for producers to bid high prices. These results are natural in the real markets.

Table 3. The results of algorithm for each unit in Case 2

GENCO	Mark-up strategy	MW dispatched	The expected pay off($)
1	0.0065	280	551
2	0.0058	260	130
3	0.0051	380	970

Case 3- This case is similar to Case 1, but the initial mark-up strategy is not considered to be zeros. The initial mark-up strategy for units 1, 2 and 3 is supposed as 0.002, 0.003, and 0.004 respectively.

The proposed algorithm is performed for this case and the obtained Nash equilibrium mark-up strategy was equal to the results of Case 1. But an important effect of this initialization was on the speed of convergence of the algorithm. The time consumed in Case 3 is about half of the required time for execution in Case 1.

Case 4- In this case, we assume that units 2 and 3 are price takers, and only unit 1 operates such as a price maker. In this case the mark up strategy of unit 1 is considered equal to the Nash mark up strategy of unit 1 in Case 1. Table 4 shows the results of this case.

Table 4. The results of algorithm for each unit in Case 4

GENCO	Mark-up strategy	MW dispatched	The expected pay off($)
1	0.005	245	225.2
2	0	280.9	-90.03
3	0	450	601.25

As shown in Table 4, the obtained pay off of unit 1 in this case is less than Case 1. This case is repeated in the same way for units 2 and 3, and in general, the results show that if only one unit chooses its price bidding higher than its marginal price, it cannot obtain greater benefits than the case in which all of them are price makers.

To identify market power, we use the market share index. Market share means the share of each producer in providing demand. In this market, because of a few numbers of units, the market shares of all units are greater than 25 percent. So we cannot identify which unit has market power ability.

5. DISCUSSION ON THE PROPOSED METHOD

The proposed algorithm can be used for hydro, thermal, nuclear and wind power plants but, the market clearing problem should include the model and constraints of each type of producer.

In the proposed algorithm, it is also possible to consider network constraints where we should consider the demand share of each load service entity (LSE), and network model. This algorithm can also be extended to GENCOs' strategic bidding in a multi period.

The complete formulation of a market clearing problem for a day-ahead energy market with regard to network constraints in a multi period scheme will be as follows:

$$
\begin{aligned}
&\max \quad \sum_{t=1}^{24}(q_{dt} * \rho_{dt}) - \sum_{t=1}^{24}\sum_{j=1}^{N_j}\sum_{b=1}^{N_B} q_{jBt}^{G} * \rho_{jBt}^{Gbid} \\
&subject\ \ to: \\
&q_{jB\min} \le q_{jBt}^{G} \le q_{jB\max} \\
&\rho_d = k1 - k2 * q_d \\
&\sum_{j=1}^{N_j}\sum_{b=1}^{N_B} q_{jBt}^{G} = q_{dt} \\
&q_{d\min} \le q_{dt} \le q_{d\max} \\
&q^{D}_{(lse)} = a\,(lse) * q_{dt} \\
&B\theta = q^{G} - q^{D} \\
&F_{\min l} \le F_l \le F_{\max l}
\end{aligned}
\tag{11}
$$

Where an (LSE) is a constant equal to the demand share of each LSE in the system and the two last equations are the dc power flow and transmission line constraints.

In the proposed algorithm, GENCOs update their mark-up strategy with arbitrary constants. But if we use the sensitivity of the market clearing price to the mark-up strategy of GENCOs for updating their mark-up strategy, the convergence of the algorithm may speed up.

6. CONCLUSION

Due to the finite number of power suppliers, the lack of enough transmission capacity, etc, the electricity markets are not fully competitive markets. Because of this, each GENCO or player in these markets should be able to choose a good bidding strategy in order to maximize its benefit.

In this paper, a new approach is proposed for presenting the best GENCO bidding strategy. The proposed method is based on the behavior of participants and the demand model, with regard to the ISO's objective function.

The test results show that demand elasticity has a major impact on GENCOs' bidding strategies. As shown in Table 1, because none of the GENCOs have a mark-up strategy equal to zero, they can obtain greater benefit than the case when they bid at their marginal cost.

As mentioned in Section 5, the method can be easily extended to consider the model of the system, where the results will be more exact and realistic. This method can be extended to consider a higher number of GENCOs and can also be used by the ISO to monitor market power.

REFERENCES

1. Ferro, R. W., Rivera, J. F. & Shahidehpour, S. M. (1998). Application of games with incomplete information for pricing electricity in deregulated power pools. *IEEE Transactions on Power Systems, 13*(1), 184-189.
2. Shahidehpour, M., Yamin, H. & Li, Z. (2002). *Market operation in electric power systems*. New York, Wiley.
3. He, Y., Song, Y. H. & Wang, X. F. (2002). Bidding strategies based on bid sensitivities in generation auction markets. *Proc. Inst. Elect.Eng.,Gen.,Transm. Dist.,149*(1), 21-26.
4. Richter, C. W. & Sheble, G. B. (1998). Genetic algorithm evaluation of utility bidding strategies for the competitive markeyplace. *IEEE Trans on Power System, 13*, 256-261.
5. Weber, J. & Overbye, T. (1999). A two-level optimization problem for analysis of market bidding strategies. *IEEE PES Summer Meeting, 2*, Edmonton, Alberta, Canada, 682-687.
6. Wen, F. S. & David, A. K. (2001). Oligopoly electricity market production under incomplete information. *IEEE Power Engineering Review*, *21*(4), 58-61.
7. Yu, Z., Sparrow, F. T., Nderitu, G., Gotham, D., Holland, F. & Morin, T. (2001). A large oligopoly competition model with an application to the Midwest electricity markets. *IEEE Power Engineering Society Summer Meeting*, *2*, Vancouver, 1040-1045.
8. Yu, Z. (2001). A strategic coalition gaming model with spatial formulation for electricity markets. *IEEE Power Review*, *21*(4), 42-44.
9. Zhang, Y. B., Luo, X. J. & Xue, J. Y. (2002). Constrained competitive Cournot equilibrium with incomplete information in electricity market. *International Conference on Power System Technology, 4*, Kunming, China, 2385-2389.
10. Wen, F. & David, A. (2001). Optimal bidding strategies and modeling of imperfect information among competitive generators. *IEEE Trans. Power Syst, 16*, 15-21.
11. David, A. & Wen, F. (2000). Strategic bidding in competitive electricity market: A literature survey. *in Proc. IEEE PowerEng. Soc. Power Eng.Conf*, Seattle, Washington USA, 2168-2173.
12. Rodriguez, C. P. & Anders, G. J. (2004). Bidding strategy design for different types of electric power market participants. *IEEE Transactions on Power System, 19*(2).
13. Li, T. & Shahidehpour, M. (2005). Strategic bidding of transmission- constrained GENCOs with incomplete information. *IEEE Transactions on Power Systems, 20*(1), 437-447.
14. Elhawary, M. E. (1995). Electrical power systems. *IEEE Press Power Systems Engineering Series*.
15. Blake, M. (2003). Game theory and electricity markets. *Drayton Analytics Research Paper Series*.
16. GAMS Release 2.50, (1999). "A User's Guide", GAMS Development Corporation, 1999 available: www.gams.com.