NONLINEAR ROBUST MODELING OF
SYNCHRONOUS GENERATORS*

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Abstract—Application of the nonlinear $\mathcal{H}_\infty$ identification method to identify a synchronous generator model is investigated in this paper. The linear $\mathcal{H}_\infty$ identification method has been well established in the literature for robust modeling despite noise and system uncertainties. Since many practical systems such as synchronous generators are nonlinear, linear models identified for particular operating conditions do not perform well for other operating conditions. To overcome this shortcoming, the linear $\mathcal{H}_\infty$ identification method has been modified to cover some nonlinearities of the systems such as saturation in synchronous machines. The derived proposed algorithm is then applied to a seventh order nonlinear model of a synchronous machine with saturation effect. In this study, the field voltage is considered as the input and the active output power and the terminal voltage are considered as the outputs of the synchronous machine. Simulation results show good accuracy of the identified models.

Keywords—Synchronous machines, identification, nonlinear systems, power system modeling, $\mathcal{H}_\infty$ identification methods

1. INTRODUCTION

As interconnected power systems have become increasingly more complex, accurate modeling and simulation of the systems have become more essential. Synchronous machines play a very important role in the stability of power systems. A proper model for synchronous machines is essential for a valid analysis of stability and dynamic performance. Almost three quarters of a century after the first publications in this area [1-2], the subject is still a challenging and attractive research topic. As the computational capability continues to grow, many advanced control strategies have been suggested for synchronous generators to improve system stability. The studies and ever-increasing size and complexity of power systems show the need for more accurate models of synchronous machines [3].

The traditional methods of modeling the synchronous machines are well specified in IEEE Standard 115 [4]. These methods assume a known structure for the synchronous machine, using well-established theories like Park’s transformation [5]. They address the problem of finding the parameters of a structure assumed to be known. Usually the procedures involve difficult and time-consuming tests. These approaches include short-circuit tests, standstill frequency response (SSFR) and open circuit frequency response (OCFR) tests. These tests can mainly be carried out when the machine is not in service. The approach is classified as white box modeling in this paper.

The main problem with the white-box modeling is that the parameters are determined individually using off-line tests. There are errors when these parameters are used collectively to simulate a synchronous generator working online. The errors may come from the fact that the assumed well-known

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structures may not accurately model the system at all operating conditions. To overcome this source of error, different structures for synchronous machines, other than the traditional dq-axis model, have been tried [6,7].

To overcome the drawbacks of the white-box modeling, identification methods based on on-line measurements have gained attention during recent years [8-21]. These methods can be divided into two categories which are classified as grey and black-box modeling in this paper.

Grey-box modeling [8-11], assumes a known structure for the synchronous machine, as the traditional methods. Then, physical parameters are estimated from on-line measurements. Although there has been some success in estimating the physical parameters from on-line measurements, the problem of wide ranges of acceptable parameters has arisen [12-13]. The main reason is that the measured on-line variables are not rich enough to adequately reflect the effect of each parameter, particularly when a high order structure for the system is considered [13]. The other problem with grey-box modeling is that the physical parameters of synchronous machines change with the operating conditions [14-16], mainly due to saturation effects and nonlinearities.

Due to the problems associated with white and grey-box modeling approaches, black-box modeling for the identification of machines has been proposed [17-20]. In black-box modeling, the structure of the model is not assumed to be known a priori. The only concern is to map the input data set to the output data set. A recent review of different nonlinear identification techniques is given in [17]. Wavelets [18], Neural networks [19-20], and Volterra series [21] are among many approaches developed for the identification of synchronous generators.

In this paper, the aim is to identify a nonlinear black-box model for a synchronous machine. Such a black-box model can be used for system analysis and controller design, especially the design of a power system stabilizer (PSS) [22]. The model can be used either in a predictive control structure for an on-line design, or as a simulator to test an off-line design.

One of the main problems associated with grey-box modeling and black-box modeling (which needs an experiment) is the measurement noise and model uncertainties. To overcome these problems for linear autoregressive (ARX) models, the H\(\infty\) identification method has been proposed [23]. Synchronous generators, however, are highly nonlinear systems. To develop a robust identification method for such a nonlinear system, in the presence of measurement noise and model uncertainties, the H\(\infty\) identification method is first generalized to cover the nonlinearities of the system. Then the developed method is applied to identify the synchronous generator.

The identification method and the nonlinear model of the system used in this paper are described in Sections II and III, respectively. The system input-output data used for model identification is presented in Section IV and the method is applied to a simulated nonlinear model of a synchronous generator. Section V concludes the paper. Some parameters of the synchronous machine have been defined in the Appendix.

2. IDENTIFICATION METHOD

The method described here is a generalization of the linear time-domain H\(\infty\) identification method proposed in [23]. In [23] a linear discrete-time system is represented by:

\[
z_k = - \sum_{i=1}^{N} a_i z_{k-i} + \sum_{i=1}^{N} b_i u_{k-i} + c_0 \theta_k
\]  

(1)

Here, a robust identification method is developed to estimate the parameters of the above model in a noisy environment.

The system under study in this paper is a synchronous generator, which is a highly nonlinear system.
The nonlinearities of the system are due to the sine and cosine functions in the state equations and the saturation effect; therefore, the linear model parameters would change dramatically with change in the operating conditions (as proved by our studies). To overcome this shortcoming, the idea of Taylor series, by which all nonlinearities such as saturation, … can be modeled, is used in this paper. The linear regression model of (1) was generalized to the nonlinear one of (2) in this paper:

\[
 z_k = -\sum_{i=1}^{n} a_{i1} z_{k-i} - \sum_{i=1}^{n} a_{2i} (z_{k-i})^2 - \cdots - \sum_{i=1}^{n} a_{Ni} (z_{k-i})^N
 + \sum_{i=1}^{m} b_{i1} u_{k-i} + \cdots + \sum_{i=1}^{m} b_{Ni} (u_{k-i})^N
 + c_0 \omega_k
\]

where \( \{a_{ij}\} \) and \( \{b_{ij}\} \) are the model parameters to be identified, \( u_k \) is the input, \( \omega_k \) is the unknown driving disturbance, \( y_k \) is the measured output, \( z_k \) is the unknown real output and \( v_k \) is the measurement noise \((y_k = z_k + v_k)\).

In (2), \( n \) is the order of the system and \( N \) is the number of nonlinear terms. A larger \( N \) results in better approximation of nonlinearities, but would increase the model complexity. If \( N=1 \) is selected, as in [23], a linear ARX model is obtained.

To identify the parameters, the above system structure can be written as:

\[
 \theta_{k+1} = A \theta_k + B \begin{bmatrix} \omega_k \\ v_k \end{bmatrix}, \quad y_k = C_k (\theta_k) \theta_k + D \begin{bmatrix} \omega_k \\ v_k \end{bmatrix}
\]

where

\[
 \theta_k = \begin{bmatrix} -a_{11} \cdots -a_{1n} \cdots -a_{N1} \cdots -a_{Nn} \\ b_1 \cdots b_m \cdots b_{N1} \cdots b_{Nm} \\ v_{k-1} \cdots v_{k-n} \end{bmatrix}
\]

and

\[
 A = \begin{bmatrix} I_{N(n+m)} & 0_{N(n+m) \times n} \\ 0_{m \times N(n+m)} & I_{n-1} \end{bmatrix}, \quad B = \begin{bmatrix} 0_{N(n+m)+n \times 1} \\ 0_{(n-1) \times 1} \end{bmatrix},
\]

\[
 C_k (\theta_k) = \begin{bmatrix} y_{k-1} \cdots y_{k-n} \cdots (y_{k-1})^N \cdots (y_{k-n})^N \\ u_{k-1} \cdots u_{k-m} \cdots (u_{k-1})^N \cdots (u_{k-m})^N \\ a_1 \cdots a_n \end{bmatrix}
\]

\[
 D = \begin{bmatrix} c_0 & 1 \end{bmatrix}
\]

The predictive model is defined as:

\[
 \hat{z}_k = -\sum_{i=1}^{n} \hat{a}_{i1} \hat{z}_{k-i} - \sum_{i=1}^{n} \hat{a}_{2i} (\hat{z}_{k-i})^2 - \cdots - \sum_{i=1}^{n} \hat{a}_{Ni} (\hat{z}_{k-i})^N
 + \sum_{i=1}^{m} \hat{b}_{i1} u_{k-i} + \cdots + \sum_{i=1}^{m} \hat{b}_{Ni} (u_{k-i})^N
\]

where \( \hat{z}_k \) is the best estimate of \( z_k \). To solve the estimation problem, the following estimation algorithm has been suggested and the stability of the algorithm, under certain conditions, has been proven in [23]:

\[
 \begin{bmatrix} \hat{\theta}_{k+1} \\ \hat{\theta}_k \end{bmatrix} = A \hat{\theta}_k + G_k \begin{bmatrix} \theta_k \\ \hat{\theta}_k \end{bmatrix} \begin{bmatrix} y_k - \hat{z}_k \\ \hat{\theta}_k \end{bmatrix}
\]

\[
 \hat{z}_k = C_k (\hat{\theta}_k) \hat{\theta}_k
\]
In the above estimator, $G_k$ is unknown. The following formulas are used to estimate the vector:

$$\hat{G}_k = \left( AQ^{-1}_k \hat{C}_k(\hat{\theta}_k) + BD \Phi_k(\hat{\theta}_k) \right)$$

(8)

Where

$$\Phi_k(\theta_k) = DD' + \hat{C}_k(\theta_k)Q^{-1}_k \hat{C}_k(\theta_k)$$

$$\hat{C}_k(\theta_k) = C_k(\theta_k) + \hat{\theta}_k'$$

$$\begin{bmatrix}
0 & 0 & 0 \\
-I & 0 & 0
\end{bmatrix}$$

Also we have

$$Q_{k+1} = \left\{ AQ^{-1}_k A' + BB' - \hat{G}_k \Phi_k(\hat{\theta}_k) G_k^T_k + \hat{\theta}_k \hat{G}_k^T + \delta_k \right\}^{-1}$$

$$- \gamma^{-2} \left[ \hat{C}_k(\hat{\theta}_{k+1}) \hat{C}_k(\hat{\theta}_{k+1}) + \delta_{k+1} \right]. \quad Q_0 > 0$$

(9)

In the above equation, $\gamma$ is the disturbance rejection factor and it should be selected as small as possible. On the other hand, it should be large enough to guarantee the convergence of the algorithm.

The next step is to assume that there is a nominal vector of parameters, $\bar{\theta}$, and a given constant $M > 0$ such that:

$$\| C_k(\theta_k) - C_k(\bar{\theta}) \| \leq M$$

Define

$$\delta_k = -AQ^{-1}_k \hat{\delta}_k \hat{G}_k^T + \hat{\delta}_k Q^{-1}_k A'$$

$$+ M \left[ \| \hat{G}_k \| A Q^{-1}_k Q^{-1}_k A' \right] + M \left[ \| \hat{G}_k \| \hat{G}_k \right]$$

(10)

$$\bar{\delta}_k = \hat{\delta}_k Q^{-1}_k \hat{C}_k(\hat{\theta}_k) + \hat{C}_k(\hat{\theta}_k) Q^{-1}_k \hat{\delta}_k + \hat{\delta}_k Q^{-1}_k \hat{\delta}_k + M^2 \| \hat{C}_k(\bar{\theta}) \| + 2M \| Q^{-1}_k \hat{C}_k(\bar{\theta}) \|$$

(11)

$\bar{\sigma}$ denotes the maximum singular value.

$$\delta_k = \hat{C}_k(\hat{\theta}_k) \hat{\delta}_k^T + \hat{\delta}_k \hat{C}_k(\hat{\theta}_k) + \frac{M}{\| \hat{C}_k(\bar{\theta}) \|} \hat{C}_k(\bar{\theta}) \hat{C}_k(\bar{\theta})$$

$$+ M^2 \begin{bmatrix}
0_{N(n+m),N(n+m)} & 0_{N(n+m),n} \\
0_{n,N(n+m)} & I_n
\end{bmatrix}$$

(12)

where $\hat{\delta}_k = \hat{C}_k(\hat{\theta}_k) - \hat{C}_k(\hat{\theta}_k)$. It is shown in [23] that the algorithm converges if and only if $Q_k$ is positive definite and

$$I - \bar{g}_k \left( Q_{k+1} + \gamma^{-2} \left[ \hat{C}_k + \left( \hat{\delta}_{k+1} + \hat{\delta}_{k+1} \right) \right] \bar{g}_k > 0$$

(13)

where $\bar{g}_k = B - G_k D$.

Using this approach, the identification method is summarized below:

a) Select a proper input signal to be applied to the system. The input signal should have a wide spectrum to cover all system dynamics. It should also have a proper magnitude.
should be large enough to cover the non-linearities and also should be small enough to be safe to perform the test.

b) Select a proper sampling time and final time (the total time for the experiment).

c) Apply the selected input signal (item a) to the system and sample the input-output data by a data acquisition system.

d) Select the order of the model \( n \) and the number of the terms \( N \) in (2). Although the real values for \( n \) and \( N \) could be very high, if a lower order can capture the required dynamics, the higher order is not preferred. In particular, for real-time control applications, a low order model, generally \( n=3 \), is sufficient.

e) Select proper values for \( \gamma \) and \( M \). The smaller value of \( \gamma \) means better disturbance rejection. The smallest value of \( \gamma \) is selected by trial and error, such that the condition in (13) is met and \( Q_k \) remains positive definite.

f) Estimate the initial seed of vector \( \theta_0^k \), \((-a_{ki}, b_{ki} \quad k=1 \cdots N, i=1 \cdots n)\), in (4) using least squares, as used in this paper, or any other identification algorithm developed for linear systems [17]. The nominal vector of parameters, \( \theta_{0} \), is the same as \( \theta_0 \).

g) Establish \( A, B, C, \theta_{0} \) and \( D \) matrices.

h) Update \( \hat{G}_k \) using (8) and calculate the other vectors given by (9)-(12) for the next iteration.

i) Update the estimated parameters \( \hat{\theta}_{k+1} \) using (7) and use the estimated value of \( G_k \) obtained in the previous step.

j) With the new estimated parameters, form the output \( \hat{y}_k \) using \( \hat{y}_k = C_k \hat{\theta}_k \hat{\theta}_k \) and compare with the measured values.

k) Go to step (h) and update the parameters, unless the parameters are converged.

The main advantage of the above algorithm is its ability to find a robust model despite noise and system uncertainties. Moreover, the algorithm can model the system nonlinearities and can be used for nonlinear systems modeling. The disadvantage of the proposed method is its relatively complicated procedure which makes it difficult to implement, compared with similar algorithms.

3. STUDY SYSTEM

A synchronous machine connected to an infinite bus through a transmission line, Fig. 1, is considered as the study system.

```
\[ y_k = C_k \hat{\theta}_k \hat{\theta}_k \]
```

Fig. 1. Structure of the study system

The nonlinear structure of synchronous generators derived in [24-25] is used to model the system. The model is described by (14) through (16).
\[ \begin{align*}
v_d &= -r_a i_d + \dot{\psi}_d - \omega \psi_q \\
v_q &= -r_a i_q + \dot{\psi}_q + \omega \psi_d \\
v_f &= r_f i_f + \psi_f \\
0 &= v_D = r_D i_D + \dot{\psi}_D \\
0 &= v_Q = r_Q i_Q + \dot{\psi}_Q 
\end{align*} \]

and

\[
\begin{bmatrix}
\omega_0 \psi_d \\
\omega_0 \psi_f \\
\omega_0 \psi_D
\end{bmatrix} = 
\begin{bmatrix}
x_d & x_{md} & x_{md} \\
x_{md} & x_f & x_{md} \\
x_{md} & x_{md} & x_D
\end{bmatrix} 
\begin{bmatrix}
-i_d \\
i_f \\
i_D
\end{bmatrix}
\]

The rotor dynamics is described by

\[
\dot{\delta} = \omega \\
\dot{\omega} = \frac{1}{J} (T_m - T_e - D_\omega)
\]

where \( T_e \), electrical torque, is usually approximated by electrical power \( (P_e) \) when the system is connected to infinite bus (which means \( \omega \approx \omega_0 \)) [24-25], i.e.:

\[
T_e \approx P_e = v_d i_d + v_q i_q
\]

In this paper, to consider the practical aspects, only the field voltage is considered as the system input and the mechanical input is considered to be constant. The field voltage is an electric signal and can be disturbed and measured more easily than the mechanical torque [10-11].

To show the strength of the identification method in a more challenging problem, the saturation effect was also considered in the model.

The first step towards the representation of saturation is to introduce the saturation factors \( K_{sd}, K_{sq} \) as [24]:

\[
x_{md} = K_{sd} x_{mdu}, x_{mq} = K_{sq} x_{mqu}
\]

where:

- \( x_{md}, x_{mq} \): unsaturated values of \( x_{md}, x_{mq} \)
- \( K_{sd}, K_{sq} \) depend on the operating conditions. They are calculated using the open circuit saturation curve (OCC). For \( K_{sd} = \frac{V_{at}}{V_{at} + \Delta V} \), \( V_{at} \) is the air-gap voltage and can be calculated by:

\[
V_{at} = |v_t + jx_i|_d
\]

\( \Delta V \) depends on the operating conditions. To model such a dependence, the most used formula is

\[
\Delta V = A e^{B(v_i - v_{t1})}
\]

where \( A, B \) and \( V_{t1} \) are proper constants which are obtained using the OCC curve.
For salient pole machines, $x_{mq}$ does not vary significantly with saturation. Therefore, $K_{sq}$ can be assumed to be equal to one for all loading conditions. For round rotors, however, $K_{sq} = K_{sq}$ is assumed. For further treatment of saturation effects refer to [24].

All variables and constants are defined in the Appendix. This model is used in the simulation studies described in this paper for the identification of the synchronous machine model.

4. SIMULATION RESULTS

To illustrate the proposed identification method, a PRBS (Pseudo Random Binary Sequence) signal is applied to the field voltage and the electric power, terminal voltage and field voltage are sampled. The sampling time was selected to be 1 ms. The input/output data collected from the system model, shown in Fig. 2, is used for the identification procedure. This data is for the operating conditions $P=0.9$ p.u., $Q=0.0$ p.u., and $v_i = 1.05$ p.u.

The identification method described in Section 2 is used to identify a synchronous machine simulated using the seventh order nonlinear model with saturation described in Section 3.

Using the first 350 samples and least squares identification method [17], the parameters of the equivalent linear model were estimated and used for $\theta_{0}^T$. The parameters $(\gamma, n, N, M)$ in the identification method were selected as $\gamma = 7$, $n = 3$, $N = 2$ and $M = 0.1$. These values were selected by trial and error to give a robust identification. In this study, the input is the sampled field voltage ($v_f$) and the outputs are the sampled active power ($P$) or terminal voltage ($v_t$), one at a time.

Identification results with the identified model and the measured variables, Fig. 2, are shown in Fig. 3. It can be seen that the proposed method is very successful in identifying the system dynamics. Since the system output and the model output are not distinguishable in Fig. 3, the error signals are shown in Fig. 4.

![Fig. 2. Data collected from the seventh order nonlinear synchronous generator model](image)

To show that the identified model has successfully covered the main non-linearities of the system, more studies were carried out. A comparison of the performance of the identified model and the system at $P = 1.1$ pu, $Q = 0.1$ pu, $v_i = 1.05$ pu is shown in Fig. 5 and the corresponding error signals are shown in Fig. 6.
Figure 6 shows that the identified model has modeled the system correctly. Additional studies, as expected, showed better performance when the operating conditions were not changed very much from the original operating conditions (at which the identification has been carried out), or when the identification algorithm is on and updates the parameters as the operating conditions change.

![Graphs of Terminal Voltage and Electrical Power](images/)

Fig. 3. Identification results with the identified model and the measured variables, Fig. 2, at $P = 0.9, Q = 0, v_r = 1.05 \text{ pu}$

![Graphs of Error Signals](images/)

Fig. 4. Error signals of Fig. 3

![Graphs of Identification Results](images/)

Fig. 5. Identification results with the identified model and the nonlinear synchronous model at $P = 1.1 \text{ pu}, Q = 0.1 \text{ pu}, v_r = 1.05 \text{ pu}$
In Table I, some of the parameters value obtained from both operating points are given. As shown in the Table, the parameters of the model do not change much with the change in operating conditions (although the saturation effects and nonlinear terms are presented in the simulation). This shows the structure proposed in this paper, by (2), can cover system nonlinearities quite accurately.

With the identifier working continuously, the performance of the identified model in response to changes in terminal voltage and power output are shown in Fig 7. In this study, the system is subjected to a relatively large change of operating conditions and the estimator updates the parameters as the operating conditions of the system change. The errors are shown in Fig. 8.

To show the importance of the nonlinear terms in the model, a linear model has been identified at one operating condition and evaluated at another. The linear model identified for an operating point performs well at the same operating condition, but when the model is verified at another operating condition, the result is not good. Fig. 9 shows the result of identifying a linear model at $P = 0.9; Q = 0; \lambda = 1.05$, but evaluating at $P = 1.1pu; Q = 0.1pu; \lambda = 1.05$. As the figure shows, a linear model is not valid if the operating conditions are changed. Comparing Fig. 9 with that of Fig. 5 the significance of nonlinear terms in modeling the outputs is quite clear.
Fig. 8. Error signals of Fig. 7

Table 1. Some parameters of the identified model in (2)

<table>
<thead>
<tr>
<th>OP</th>
<th>$a_{11}$</th>
<th>$a_{12}$</th>
<th>$b_{11}$</th>
<th>$b_{12}$</th>
<th>$v_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>.6568</td>
<td>-.1165</td>
<td>-10.116</td>
<td>17.713</td>
<td>.313e-4</td>
</tr>
<tr>
<td>2nd</td>
<td>.6481</td>
<td>-.1154</td>
<td>-10.606</td>
<td>17.577</td>
<td>.299e-4</td>
</tr>
</tbody>
</table>

Modeling terminal voltage

<table>
<thead>
<tr>
<th>OP</th>
<th>$a_{11}$</th>
<th>$a_{12}$</th>
<th>$b_{11}$</th>
<th>$b_{12}$</th>
<th>$v_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>.6407</td>
<td>-.1146</td>
<td>-11.44</td>
<td>17.218</td>
<td>.399e-4</td>
</tr>
<tr>
<td>2nd</td>
<td>.6416</td>
<td>-.1143</td>
<td>-11.55</td>
<td>17.218</td>
<td>.371e-4</td>
</tr>
</tbody>
</table>

* OP stands for the operating point

Fig. 9. Identification results with the identified model and the linear synchronous model at $P = 1.1\, \text{pu}; Q = 0.1\, \text{pu}; v = 1.05\, \text{pu}$

5. CONCLUSION

Nonlinear identification of a synchronous generator using the nonlinear $H\infty$ identification method is described in this paper. The proposed method is classified as black-box modeling. In this paper, the algorithm was first modified to cover the nonlinearities of nonlinear systems such as saturation effects in synchronous generators. The proposed method has been tested on a 7th order nonlinear simulated model of synchronous machines.
Simulation results show that the proposed method can be used successfully for the identification of a nonlinear synchronous machine model. The obtained black-box model can be used for system analysis and for designing a power system stabilizer (PSS) in a predictive on-line control structure.

The proposed method requires only a small perturbation of the field voltage, and all required signals are easily measurable. Based on this, it seems feasible of application for the identification of a large range of synchronous machines.

REFERENCES


**APPENDIX**

The main variables and constants of (14) are:

- $i = \frac{\sqrt{P^2 + Q^2}}{v_i}, \varphi = \tan^{-1} \frac{Q}{P}$
- $v_d = v_i \sin \delta, v_q = v_i \cos \delta$
- $i_d = i \sin(\delta + \varphi), i_q = i \cos(\delta + \varphi)$
- $v_{bd} = v_d - r_d i_d + x_d i_q, v_{bq} = v_q - r_q i_q - x_q i_d$
- $v_g = \sqrt{v_{bd}^2 + v_{bq}^2}$
- $x_d = x_{ad} + x_{tq}, x_q = x_{aq} + x_{f}$
- $J, D$ rotor inertia and damping factor
- $x_l$ stator leakage reactance
- $x_{md}, x_{mq}$ direct and quadrature axis mutual reactances
- $x_d'$ direct transient reactance
- $x_s, R_t$ line and transformer reactance and resistance
- $\delta$ rotor angle
- $\omega$ rotor speed
- $T_m$ mechanical input torque
- $P, Q$ terminal active and reactive power per phase
- $v_i$ terminal voltage
- $v_B$ infinite bus voltage
- $i_f, v_f$ field current and voltage
- $r_f, x_f$ field resistance and reactance
- $\Psi_d, \Psi_q$ direct and quadrature axis damper winding fluxes
- $\Psi_{d1}, \Psi_{q1}$ direct and quadrature axis stator fluxes
- $\Psi_f$ field winding flux
- $i_{D1}, i_{Q1}$ direct and quadrature axis damper currents
- $v_{D}, v_{Q}$ direct and quadrature axis damper voltages
- $r_{D}, r_{Q}$ direct and quadrature axis damper resistances
- $x_{D}, x_{Q}$ direct and quadrature axis damper reactances