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THE PRINCIPAL COMPONENT INVERSE ALGORITHM FOR DETECTION IN THE PRESENCE OF REVERBERATION USING AUTOREGRESSIVE MODEL*

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Abstract– Reverberation noise in active sonar leads to a very complicated situation for target detection. Reverberation is often modeled as the autoregressive model. In this paper, the autoregressive model is considered for reverberation and the Principal Component Inverse (PCI) algorithm is used to separate target echo signal from reverberation. This consideration helps us to propose a new method to improve computational complexity for the rank determination of the observation matrix via singular value decomposition. It is shown that this new method is efficient on real data to separate target echo signal from reverberation.

Keywords– Reverberation cancellation, autoregressive model, principal component inverse (PCI) algorithm

1. INTRODUCTION

Target detection in the presence of reverberation for active sonar (especially in shallow water) is an important problem in underwater acoustic signal processing. Reverberation is caused mainly by the multiple reflections/diffusions/diffractions of the transmitted signal by the surface and bottom interfaces. Since the reverberation is strongly correlated with the signal, classical detection methods like matched filtering (MF) are inefficient [1]. Many works have been presented to reduce reverberation effect and improve target detection. Here, we present a number of them.

A model is often considered for reverberation to improve detection. Some models have been introduced in [2-6]. Each introduced model in [2-6] uses some properties of reverberation to improve target detection. In this paper, we have considered the properties of reverberation using two of these models simultaneously and then solve the problem of reverberation cancellation from the received signal.

In [2], a statistical model was considered for reverberation as nonstationary, colored noise. This model leads to elaborate detection algorithms which normalize and whiten reverberation. In [2], the prewhitener is based on an autoregressive model for the reverberation. At the output of the prewhitener a generalized likelihood ratio test detector is implemented. It was shown that algorithms based on this approach had some problems when the Doppler shifts of reverberation and target echo are similar. To improve this problem, reverberation was considered as a sum of undesirable echoes in [3]. The method for detection consists of estimating these echoes and deleting them before applying the classical MF. It is important to choose a metric to distinguish reverberation echoes from target echoes, and since the target echo power is often lower than reverberation power, echo power has been chosen as a metric and an algorithm which is able to separate echoes with different power has been used. This algorithm is the Principal Component Inverse (PCI) algorithm which has been introduced in [4] and [5]. This algorithm originally assumes that noise is completely different from the searched signal, but in [3], it has been

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shown that PCI can be applied to detection in presence of reverberation. In particular, the importance of the rank of the observation matrix has been shown. PCI is a more robust method with regard to reverberation properties.

In [1], a new algorithm Signal Subspace Extraction (SSE) based on this more real-life model has been presented. The SSE algorithm divides the reverberation with target echo into three parts: higher reverberation echoes, the target echo and lower reverberation echoes.

It makes use of a low rank characteristic of target echoes subspace, and separates the signal subspace via the singular value decomposition (SVD) method.

In this paper, the autoregressive model is considered for reverberation and the Principal Component Inverse (PCI) algorithm is used to separate target echo signal from reverberation. This consideration helps us to propose a new method to improve computational complexity of the rank determination of the observation matrix via singular value decomposition. So, the paper is arranged as follows:

Section 2 quickly reviews the classical detection/testing hypothesis, the Block Normalized Matched Filter (BNMF), the whitening BNMF. Section 3 presents the PCI algorithm, and analyzes the critical point of matrix rank estimation. Section 4 presents the new algorithm for matrix rank estimation. In section 5, we give the results of reverberation reduction via this new algorithm using an example with real reverberation data and verify the conclusions.

2. DETECTION PROBLEM IN PRESENCE OF REVERBERATION

Let $e(t)$ be the transmitted signal with duration T. Now, the detection problem is

$$\begin{cases} H_0 : x(t) = n(t) + b(t) \\ H_1 : x(t) = s(t) + n(t) + b(t) \end{cases} \tag{1}$$

where $n(t)$ is the reverberation noise, $b(t)$ is the white ambient noise, and $s(t)$ is the target echo. Noises b and n are assumed Gaussian and independent. $s(t)$ differs from $e(t)$ to a time delay τ, a doppler shift f_D and an amplitude attenuation A:

$$s(t) = Ae(t-\tau)\exp(2i\pi f_D t) \tag{2}$$

All signals are complex valued and represent the sonar output after complex demodulation. We work with time sampled signals. Let x_t be the sampled vector of $x(t)$ and $\bar{s}_t$ be the sampled normalized signal

$$\bar{s}_t(\tau, f_D) = Ae_{t-\tau}\exp(2i\pi f_D t) \tag{3}$$

In sonar, the problem is not only to choose between H_0 and H_1 but also to estimate the attenuation A, the delay τ, and the Doppler frequency f_D. We use the classical generalized likelihood ratio test (GLRT) to build the different algorithms. As the reverberation n is nonstationary, its power is time varying. We propose building a block-by-block detector. The received signal is divided into blocks of length N (N is the transmitted signal length) which are shifted by d (shift of window) samples. The procedure allows us to obtain a set $\{x_t^i\}$ (for block i) of length N. The statistic test of the BNMF is

$$L_i(f_D) = \frac{\left|\sum_{t=0}^{N-1} \bar{s}_t^*(f_D) x_t^i\right|^2 \Big/ \sum_{t=0}^{N-1} \left|\bar{s}_t(f_D)\right|^2}{\left(1/2N\right)\sum_{t=0}^{N-1}\left|x_t^i\right|^2} \tag{4}$$

$L_i(f_D)$ is computed on each block $\{x_t^i\}$ and for different Doppler shifts $f_D = k\Delta f$. The parameter Δf is sampling rate for the estimation of f_D. This measures the precision of the Doppler shift estimation. Let *M* be the number of blocks and *K* the number of Doppler samples. The BNMF algorithm allows one to obtain a matrix $\{L_i(k\Delta f)\}_{i,k}$. Hypothesis H_1 is chosen if $\max_{i,k} L_i(k\Delta f)$ is larger than a given threshold η . In addition, this maximum estimates the block containing the target echo and the corresponding Doppler frequency.

As reverberation is often considered to be colored noise, it is obvious that a whitening step will improve detection. We propose a detector be built from the BNMF and the whitening proposed by Carmillet [6]. The nonstationarity of reverberation involves developing an adaptive algorithm. In the first block assumed without target, reverberation is estimated by means of an AR model. In the following block, data are whitened by the corresponding MA (moving average) filter, and the BNMF is then applied on whitened data. This algorithm is called whitening BNMF [7].

Simulations have shown that a whitening procedure is inefficient when target echo and reverberation echoes have similar Doppler properties. This situation appears most often when the transmitted signal is a hyperbolic frequency-modulated (HFM) signal because this signal is unaffected by Doppler, so reverberation and target echo have similar Doppler properties. This statistical model does not take into account the link between reverberation and transmitted signal and so we propose a combination of this model and another reverberation model which takes advantage of the connection between similar Doppler properties of the target and reverberation echoes.

3. THE PCI ALGORITHM

Let us consider now the reverberation as a sum of echoes from the transmitted signal. The method is based on the power contrast between reverberation echoes and target echo. The PCI algorithm [1, 3], which is a particular case of subspace methods, is used to separate reverberation echoes and the target echo by estimating the reverberation subspace. This algorithm was originally developed to detect a weak signal in high interference which is assumed to be different from the transmitted signal.

Modeling reverberation as a sum of echoes issued from the transmitted signal implies that reverberation and the target echo have almost the same properties. A metric and a hypothesis are then needed in order to separate them. Power is used as a metric because we assume that reverberation echoes power is much greater than both signal and white noise power. Because of this assumption, the PCI algorithm estimates the reverberation subspace. As a consequence, the reverberation echoes can be deleted and classical treatments (like MF) can be used. The subspace methods require a generating matrix. Kumaresan et al. [4] proposed a matrix built from the received time signal by cutting x into blocks X_i. This matrix, denoted by $\mathbf{Y}_i^k$ on block *i*, is called the forward matrix

$$\mathbf{Y}_i^k = \begin{bmatrix} x_i(k) & x_i(k-1) & \dots & x_i(1) \\ x_i(k+1) & x_i(k) & & x_i(2) \\ \vdots & & & \\ x_i(l) & x_i(l-1) & & x_i(l-k+1) \end{bmatrix} \tag{5}$$

where *l* is the block length and *k* is chosen close to *l*/2. The choice of l will be discussed in the next section. The PCI algorithm consists of decomposing $\mathbf{Y}_i^k$ into two matrices $\mathbf{Y}_i^r$ and $\mathbf{Y}_i^o$

$$\mathbf{Y}_i^k = \mathbf{Y}_i^r + \mathbf{Y}_i^o \tag{6}$$

where $\mathbf{Y}_i^r$ spans the reverberation subspace and $\mathbf{Y}_i^o$ the signal plus white noise subspace. As reverberation echoes are assumed to be stronger than the target echo and white noise, $\mathbf{Y}_i^r$ is built with the largest singular values of $\mathbf{Y}_i^k$. Actually, if *r* is the reverberation subspace rank, $\mathbf{Y}_i^r$ is the best *r*-rank approximation of $\mathbf{Y}_i^k$ and is obtained via the Singular Value Decomposition (SVD) of $\mathbf{Y}_i^k$ and by the Eckart and Young [1, 8] theorem:

$$\mathbf{Y}_i^k = \mathbf{U}\Sigma\mathbf{V}^H = \left[\mathbf{U}^r \middle| \mathbf{U}^o\right]\begin{bmatrix}\mathbf{\Sigma}^r & \mathbf{0} \\ \mathbf{0} & \mathbf{\Sigma}^o\end{bmatrix}\left[\mathbf{V}^r \middle| \mathbf{V}^o\right]^H = \mathbf{U}^r\mathbf{\Sigma}^r\mathbf{V}^{rH} + \mathbf{U}^o\mathbf{\Sigma}^o\mathbf{V}^{oH} = \mathbf{Y}_i^r + \mathbf{Y}_i^o \tag{7}$$

where $\mathbf{U}$ is the left singular vector matrix of $\mathbf{Y}_i^k$, $\mathbf{V}$ is the right singular vector matrix of $\mathbf{Y}_i^k$, and Σ a diagonal matrix which contains the decreasing singular values of $\mathbf{Y}_i^k$, $\{\sigma_i\}(\sigma_1 > \sigma_2 > ...)$. A vector X_i^r is then collected from $\mathbf{Y}_i^r$. The subspace reverberation estimation is made for all the blocks in order to obtain a vector X^r which contains only reverberation. Finally, the detection processing is done on the vector $X - X^r$ and this vector only includes white noise and target echo (if occur). Two assumptions are necessary for a correct running of PCI:

• *s* and *b* must be less powerful than *n*;

• the rank *r* of $\mathbf{Y}_i^r$ must be small.

The first hypothesis is used to separate the reverberation subspace and the signal subspace [see (9)]. The second hypothesis is necessary to obtain good separation. The rank of one or several echoes is often unknown. The rank estimation is the most difficult step of the algorithm, and very briefly has been described by Tufts and Kirsteins [7]. The rank is linked to the received signal power. It is estimated by computing the sum of the singular values of $\mathbf{Y}_i^k$ and comparing it to a threshold *P* which is linked to our prior knowledge of the target echo and white noise power. If the sum is not greater than *P*, PCI does not treat block X_i. If there exists an index *M* as follows ($R_{\mathbf{Y}}$ is the rank of $\mathbf{Y}_i^k$):

$$\sum_{i=0}^{M} \sigma_{R_Y - i}^2 > P \tag{8}$$

then PCI is applied to block X_i and *r* is equal to $R_Y - M + 1$. For setting up threshold for the procedure the sum of squares of all singular values under hypothesis H_0 (no signal case) is necessary. As it has been shown in [10], we need a mixture χ^2 distribution to calculate threshold *P*. Here, we are looking for a new method that finds threshold in a simpler way than before. So, an autoregressive (AR) model for the reverberation and using PCI to separate reverberation signal are considered.

4. COMBINATION OF THE PCI ALGORITHM WITH THE AR MODEL

In [2], an autoregressive model for the reverberation has been shown, through analysis of real data, to be an accurate model. Let $e(t)$ be the transmitted signal with duration *T*. Now, the detection problem is

$$\begin{cases} H_0 : x(t) = n(t) \\ H_1 : x(t) = s(t) + n(t) \end{cases} \tag{9}$$

Where *n*(*t*) is the reverberation noise, and *s*(*t*) is the target echo. The noise *n*, is modeled as a Gaussian AR process of order *p*

$$n(t) = \sum_{k=1}^{p} a_k n(t-k) + b(t) \tag{10}$$

Where *b*(*t*) is zero-mean complex white Gaussian noise with variance σ^2. Let *k*=*p*+1 in matrix $\mathbf{Y}_i^{p+1}$, and define **B**' as follows:

$$\mathbf{B}'=\begin{bmatrix} x_i(p+1)-b(p+1) & x_i(p) & \ldots & x_i(1) \\ x_i(p+2)-b(p+2) & x_i(p+1) & & x_i(2) \\ \vdots & & & \\ x_i(l)-b(l) & x_i(l-1) & & x_i(l-p) \end{bmatrix}=\mathbf{Y}_i^{p+1}+\mathbf{E} \tag{11}$$

where E is

$$\mathbf{E}=-\begin{bmatrix} b(p+1) & 0 & \cdots & 0 \\ b(p+2) & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ b(l) & 0 & \cdots & 0 \end{bmatrix}=\left[-\mathbf{b}\middle|\mathbf{0}\,\mathbf{0}\cdots\mathbf{0}\right] \tag{12}$$

Let $\beta_1' \geq \beta_2' \geq \cdots \geq \beta_{p+1}'$ and $\alpha_1' \geq \alpha_2' \geq \cdots \geq \alpha_{p+1}'$ denote the ordered singular values of **B**' and $\mathbf{Y}_i^{p+1}$, respectively. From (11), the first column of B' is a linear combination of the last *p* linearly independent columns. Thus, the rank of **B**' is *p* and β_{p+1}'=0. From the perturbation theory in [11, 12], it can be shown that

$$\alpha_{p+1}' \leq \varepsilon_1 = \|\mathbf{b}\|_2 \leq \sqrt{c(l-p)}\sigma = \varepsilon_L \tag{13}$$

where ε_1 is largest SV of **E**, $\|\mathbf{b}\|_2$ is 2-norm of **b**, and *c*(*l*-*p*) represents the percentile value of the Chi-Square distribution ($\chi^2_{1-\alpha}$) with *l*-*p* degree of freedom (from theory of statistical significance testing and bounds on the 2 norm of matrix, for a given level of significance α , ε_1 is bounded by ε_L [11]). Thus, if $\alpha_p' \geq \varepsilon_L \equiv \delta$, then $\alpha_{p+1}' \leq \delta \leq \alpha_p'$ and the order of the AR model can be derived correctly. In fact, the effective rank of reverberation subspace in matrix $\mathbf{Y}_i^{p+1}$ is identical to the order of the AR model *n*(.).
The value of ε_L depends on the variance of noise σ^2, which in practice is unknown. However, the variance of noise σ^2 can be estimated from residual error of least squares solution (which is denoted by $S^2(q)$) given by Karimi in [13] as follows:

$$\hat{\sigma}^2=\frac{S^2(k)}{(1-\frac{k}{N-k})} \qquad ;k \geq p \tag{14}$$

where *k* is the supposed order for the AR model.

The following algorithm for rank determination of reverberation subspaces is useful as follows:

1) Consider an autoregressive model for the reverberation.
2) Suppose that *x* is the vector of received time signal. Under hypothesis H_0 (no signal case), create a matrix from the received time signal by cutting *x* into blocks X_i. This matrix, noted $\mathbf{Y}_i^k$ on block *i*, is defined above in (5) and *k* is chosen to be larger than the estimated order.
3) Find the singular values $\alpha_1', \alpha_2', \cdots, \alpha_k'$ of $\mathbf{Y}_i^k$.
4) Use (14) to obtain an estimate of the variance of noise σ^2.
5) Obtain $\delta = \varepsilon_L$ in equation (13) to determine *r* such that $\alpha_{r+1}' \leq \delta \leq \alpha_r'$.
6) Repeat 2-5 with *k*=*r+1*, to find new rank *r'* of $\mathbf{Y}_i^k$.
   If *r*'<*r* and *r*'>1, then set *r*=*r*' and repeat 6, else the order is equal to *r*.
7) A vector X_i^r is then collected from $\mathbf{Y}_i^r = \mathbf{U}^r\mathbf{\Sigma}^r\mathbf{V}^{rH}$. The subspace reverberation estimation is made for all the blocks in order to obtain a vector X^r which contains only reverberation.
8) Finally, the vector $X - X^r$ is constructed. This vector encompasses noise and target echo and rejects reverberation.

5. RESULTS WITH REAL DATA

Now, we apply this algorithm to real data, and verify the effect of this algorithm in reverberation cancellation. It is important to know that the Doppler shift of target echo and reverberation are similar. Let us show an example of real surface and bottom reverberation. The transmitted signal is continuous wave (CW). The frequency is 50 000 Hz, the transmitted pulse-width is 3 msec. After receiving signal, the sampling frequency of the received signal is 200 000 samples per second. Figure 1 shows a target echo between 40-50 meters along with the surface and bottom reverberation. As it is shown in Fig.1 the reverberation lasts much longer than the transmitted signal and is more powerful than target echo. We divide the received signal into blocks with 1200 samples, each block has 600 samples that overlap with the former block. Then, we apply the procedures 1-8 defined in section 4 to reject the reverberation from the received signal. Finally, for each processed block, the vector $X - X^r$ is constructed and plotted in Fig.2. As it is seen in Fig. 2, the target echo is encompassing and the reverberation is rejected.

We also plot the received signal when target is placed at 73m and 90m from hydrophone in Fig. 3 and Fig. 5, respectively. The processed signals of these two cases with the above algorithm are depicted in Fig. 4 and Fig. 6. It has been shown from these figures that as target echo becomes lower than reverberation the algorithm works more efficiently.

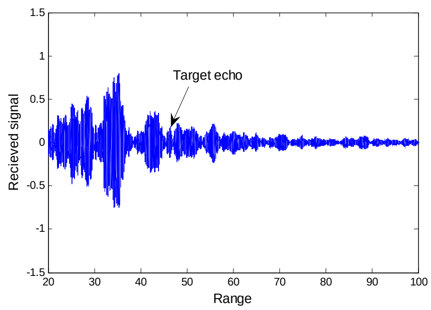

Fig.1. Received signal when target echo is at 45m

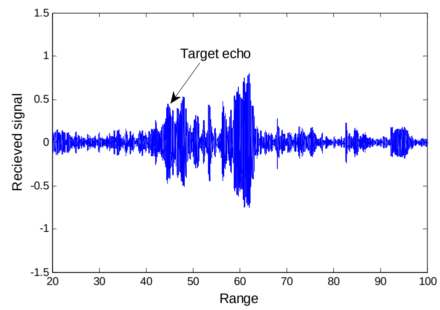

Fig. 2. Reverberation cancellation from the received signal in Fig.1

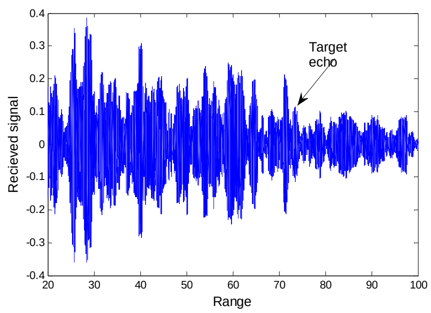

Fig. 3. Received signal when target echo is at 73m

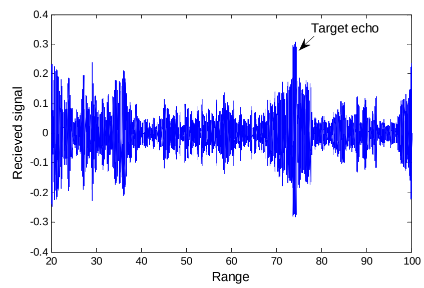

Fig. 4. Reverberation cancellation from the received signal in Fig. 3

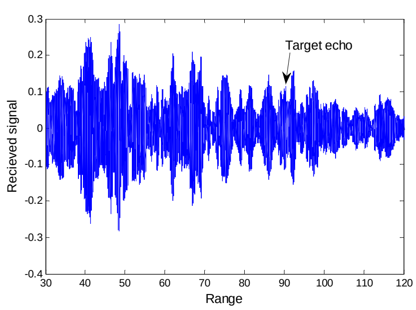

Fig. 5. Received signal when target echo is at 90m

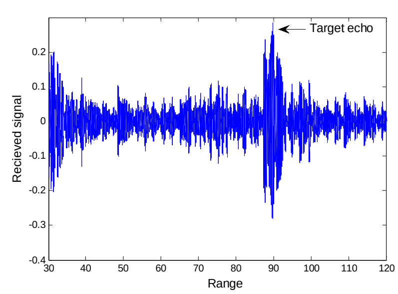

Fig. 6. Reverberation cancellation from the received signal in Fig. 5

6. CONCLUSION

The autoregressive model is considered for reverberation and the Principal Component Inverse (PCI) algorithm is used to separate target echo signal from reverberation. This consideration enables us to propose a new method to improve computational complexity for the rank determination of the observation matrix via singular value decomposition.

We apply this algorithm to real data, and verify the effect of this algorithm in reverberation cancellation when the Doppler shift of target echo and reverberation are similar. It is shown that this new method is efficient on real data to separate target echo signal from reverberation, particularly when target echo has lower power than reverberation.

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